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## Bloom-Gilman duality of the nucleon structure function and the elastic peak contribution<sup>a</sup>

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### Abstract

The occurrence of the Bloom-Gilman duality in the nucleon structure function is investigated by analyzing the  $Q^2$ -behavior of low-order moments, both including and excluding the contribution arising from the nucleon elastic peak. The Nachtmann definition of the moments has been adopted in order to cancel out target-mass effects. It is shown that the onset of the Bloom-Gilman duality occurs around  $Q^2 \sim 2 (GeV/c)^2$  if only the inelastic part of the nucleon structure function is considered, whereas the inclusion of the nucleon elastic peak contribution leads to remarkable violations of the Bloom-Gilman duality.

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The investigation of inelastic lepton scattering off nucleon (and nuclei) can provide relevant information on the concept of parton-hadron duality, which deals with the relation among the physics in the nucleon-resonance and Deep Inelastic Scattering (*DIS*) regions. As is well known, well before the advent of *QCD*, local parton-hadron duality was observed empirically by Bloom and Gilman [1] in the proton structure function  $F_2(x, Q^2)$  measured at *SLAC* (where  $x \equiv Q^2/2m\nu$  is the Bjorken scaling variable,  $m$  the nucleon mass and  $Q^2$  the squared four-momentum transfer). More precisely, they found that the smooth scaling curve measured in the *DIS* region at high  $Q^2$  represents a good average over the resonance bumps seen in the same  $x$  region at low  $Q^2$ .

In Ref. [2] we addressed the specific question whether and to what extent the Bloom-Gilman duality already observed in the proton occurs also in the structure function of a nucleus. To that end all the available experimental data for the structure functions of proton, deuteron and light complex nuclei were analyzed in terms of low-order moments in the  $Q^2$  range from 0.3 to 5  $(\text{GeV}/c)^2$ . If only the inelastic parts of the structure functions are considered, we found that in case of the proton and the deuteron the Bloom-Gilman duality is fulfilled starting from  $Q^2 \sim 2 (\text{GeV}/c)^2$ , whereas in case of complex nuclei, despite the poor statistics of the available data, the onset of the local parton-hadron duality is clearly anticipated. Besides these interesting findings, we observed also that the inclusion of the contribution arising from the nucleon elastic peak leads to remarkable violations of the local parton-hadron duality for all the targets considered. In this contribution we present an improvement of the work of Ref. [2] about the failure of local parton-hadron duality around the nucleon elastic peak.

Following the works of Refs. [3, 4], the analysis of Ref. [2] was carried out using the Cornwall-Norton definition of the moments, viz.

$$M_n^{(CN)}(Q^2) \equiv \int_0^1 d\xi \xi^{n-2} F_2(\xi, Q^2) \quad (1)$$

where  $\xi \equiv 2x/[1 + \sqrt{1 + 4m^2x^2/Q^2}]$  is the Nachtmann variable. The  $Q^2$  behavior of Eq. (1) was compared with the one of the moments  $A_n^{(CN)}(Q^2)$  of the leading-twist (dual) structure function, viz.

$$A_n^{(CN)}(Q^2) \equiv \int_0^1 d\xi \xi^{n-2} F_2^{(dual)}(\xi, Q^2) \quad (2)$$

with

$$\begin{aligned} F_2^{(dual)}(\xi, Q^2) &= \frac{x^2}{(1 + \frac{4m^2x^2}{Q^2})^{3/2}} \frac{F_2^{(LT)}(\xi, Q^2)}{\xi^2} \\ &+ 6 \frac{m^2}{Q^2} \frac{x^3}{(1 + \frac{4m^2x^2}{Q^2})^2} \int_\xi^1 d\xi' \frac{F_2^{(LT)}(\xi', Q^2)}{\xi'^2} \\ &+ 12 \frac{m^4}{Q^4} \frac{x^4}{(1 + \frac{4m^2x^2}{Q^2})^{5/2}} \int_\xi^1 d\xi' \int_{\xi'}^1 d\xi'' \frac{F_2^{(LT)}(\xi'', Q^2)}{\xi''^2} \end{aligned} \quad (3)$$

where the  $\xi$ -dependence as well as the various integrals appearing in the r.h.s. account for target mass effects, which have to be included in order to cover the low  $Q^2$  region [3]. In Eq. (3)  $F_2^{(LT)}(x, Q^2)$  represents the leading-twist ( $LT$ ) nucleon structure function, fitted to high  $Q^2$  proton and deuteron data, and extrapolated down to low values of  $Q^2$  by means of the Altarelli-Parisi evolution equations. In the  $DIS$  region one gets  $F_2^{(LT)}(x, Q^2) = \sum_f e_f^2 x [\rho_f(x, Q^2) + \bar{\rho}_f(x, Q^2)]$ , with  $\rho_f(x, Q^2)$  being the quark distribution of flavor  $f$ .

However, Eqs. (1-3) suffer from a well known mismatch, because  $\xi(x=1) \equiv \xi_{max} = 2/(1 + \sqrt{1 + 4m^2/Q^2}) < 1$ . Therefore, while the evaluation of the integral of Eq. (1) stops at the physical threshold  $\xi_{max} < 1$ , corresponding to the elastic end-point  $x = 1$ , the r.h.s. of Eq. (2) requires the values of the dual function (3) in the unphysical region  $\xi_{max} \leq \xi \leq 1$ . Consequently, the comparison of the experimental  $M_n^{(CN)}(Q^2)$ , obtained using all the available data sets and including the nucleon elastic peak contribution, with the dual moments  $A_n^{(CN)}(Q^2)$  should be handled with care. Since the mismatch originates from target-mass corrections (i.e., from the fact that  $m \neq 0$ ) we now adopt a different definition of the moments, which avoids completely the mismatch problem, namely we consider the Nachtmann definition of moments [5], given by

$$M_n(Q^2) \equiv \int_0^1 dx \frac{\xi^{n+1}}{x^3} F_2(x, Q^2) \frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \quad (4)$$

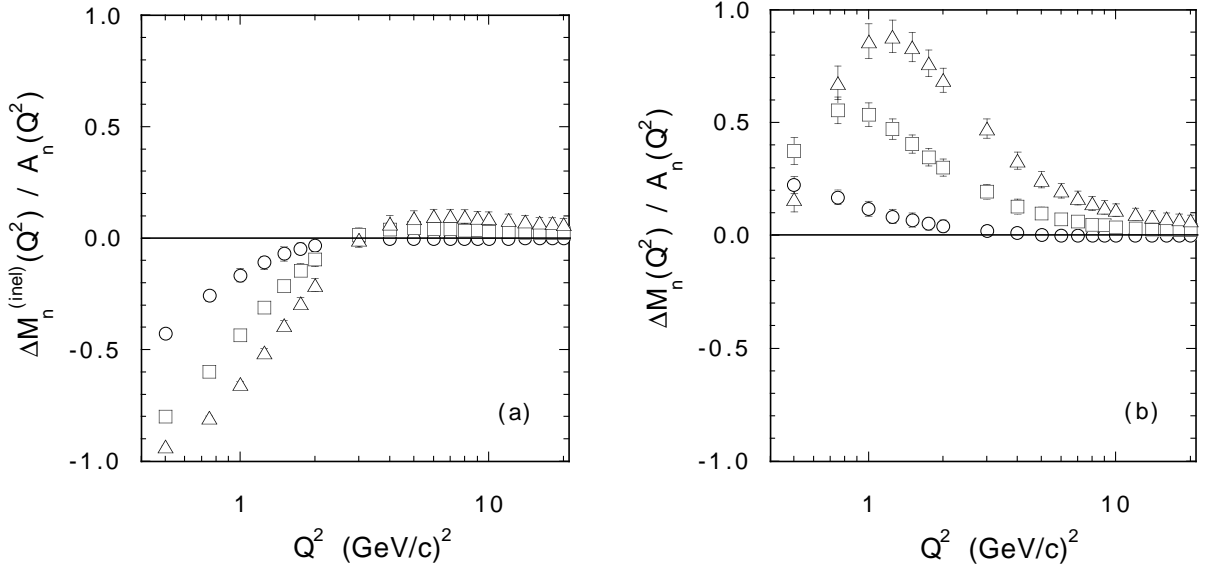
where  $r \equiv \sqrt{1 + 4m^2x^2/Q^2}$  ( $\xi = 2x/(1+r)$ ). Since the r.h.s. of Eq. (4) projects out the contributions of spin- $n$  operators [5], the  $Q^2$  behavior of  $M_n(Q^2)$  has to be compared with the one of the  $LT$  moments  $A_n(Q^2)$ , defined as

$$A_n(Q^2) \equiv \int_0^1 dx x^{n-2} F_2^{(LT)}(x, Q^2) \quad (5)$$

where the  $LT$  structure function  $F_2^{(LT)}(x, Q^2)$  does not include any target-mass corrections by definition. Now, the evaluation of Eq. (5) requires the values of the structure function  $F_2^{(LT)}(x, Q^2)$  only in the physical region  $0 \leq x \leq 1$ .

As pointed out in Ref. [4] and described in Ref. [2], the Bloom-Gilman duality should manifest itself in the dominance of the  $LT$  moments  $A_n(Q^2)$  in the  $Q^2$  behavior of low-order moments  $M_n(Q^2)$  starting from a value  $Q^2 \simeq Q_0^2$  almost independent of the order  $n$  of the moment (for high values of  $n$  the moments of the structure functions  $F_2(x, Q^2)$  and  $F_2^{(LT)}(x, Q^2)$  should differ because of the rapidly varying behavior of the nucleon-resonances peaks). In other words we expect that the low-order residual moments  $\Delta M_n(Q^2) \equiv M_n(Q^2) - A_n(Q^2)$  become a small fraction of the  $LT$  moments  $A_n(Q^2)$  for  $Q^2 \gtrsim Q_0^2$ . This is exactly what we see for  $Q^2 \gtrsim Q_0^2 \sim 2 \text{ (GeV/c)}^2$  in Fig. 1(a), where some low-order residual moments  $\Delta M_n(Q^2)$  have been evaluated including only the inelastic data. However, when the contribution from the nucleon elastic peak is added, the dominance of  $A_n(Q^2)$  starts from values of  $Q^2$  which strongly depend upon  $n$  for  $n > 2$ . As for  $n = 2$ , since the second moment  $M_2(Q^2)$  corresponds to the area of the structure function, the dominance of the  $LT$  second moment  $A_2(Q^2)$  correspond to the *global* parton-hadron duality, which holds with and without the

inclusion of the nucleon elastic peak contribution (see Fig. 1). Our results imply that the parton-hadron duality holds for local averages of the nucleon structure function over the resonance bumps, but the elastic peak. Note that the applicability of the concept of local parton-hadron duality in the region around the nucleon elastic peak was found to be critical also in Refs. [1, 4]. Finally, let us point out that results of the same quality as those shown in Fig. 1 hold as well in case of the deuteron structure function.



**Figure 1.** Ratio of the residual moments  $\Delta M_n(Q^2) \equiv M_n(Q^2) - A_n(Q^2)$  to the  $LT$  moments  $A_n(Q^2)$  (see Eqs. (4-5)) vs.  $Q^2$ . Dots, squares and triangles correspond to  $n = 2, 4$  and  $6$ , respectively. For the calculation of  $A_n(Q^2)$  (Eq. (5)) the parton distributions of Ref. [6] have been adopted. In (a) only the inelastic part of the proton structure function is considered, whereas in (b) also the proton elastic peak contribution is included in the determination of Eq. (4). For the experimental data set adopted for the evaluation of Eq. (4) see Ref. [2].

In conclusion, the occurrence of the Bloom-Gilman duality in the nucleon structure function has been investigated by analyzing the  $Q^2$ -behavior of low-order moments, both including and excluding the contribution arising from the nucleon elastic peak. The Nachtmann definition of the moments has been adopted in order to cancel out target-mass effects. It has been shown that the onset of the Bloom-Gilman duality occurs around  $Q^2 \sim 2 \text{ (GeV/c)}^2$  if only the inelastic part of the nucleon structure function is considered, whereas the inclusion of the nucleon elastic peak contribution leads to remarkable violations of the Bloom-Gilman duality.

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